Maths (Basic) Delhi (Set 1)

General Instructions :

(i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory:

(ii) Section A : Q. No. 1 to 20 comprises of 20 questions of one mark each.

(iii) Section B : Q. No. 21 to 26 comprises of 6 questions of two marks each.

(iv) Section C: Q. No. 27 to 34 comprises of 8 questions of three marks each.

(v) Section D : Q. No. 35 to 40 comprises of 6 questions of four marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to attempt only one of the choices **in such questions.**

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

Question: 1

HCF of 144 and 198 is

(a) 9

(b) 18

(c) 6

(d) 12

Solution:

Using Euclid's division algorithm,

$198 = 144 \times 1 + 54$

$144 = 54 \times 2 + 36$

$54 = 36 \times 1 + 18$

$36 = 18 \times 2 + 0$

$\Rightarrow$HCF of 144 and 198 is 18.

Hence, the correct answer is option (b).

Question: 2

The median and mode respectively of a frequency distribution are 26 and 29. Then its mean is

(a) 27.5

(b) 24.5

(c) 28.4
(d) 25.8

Solution:

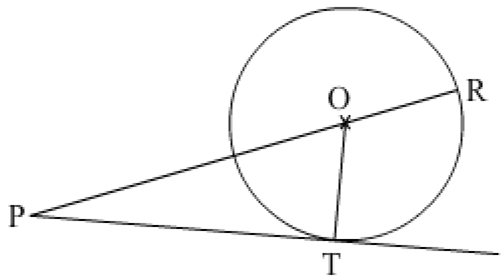
The empirical relationship between mean, median and mode is

$$\text{Mode} = 3\text{Median} - 2\,\text{Mean}$$
$$\Rightarrow 2\text{Mean} = 3\text{Median} - \text{Mode}$$
$$\Rightarrow 2\text{Mean} = 3 \times 26 - 29$$
$$\Rightarrow \text{Mean} = \frac{49}{2} = 24.5$$

Hence, the correct answer is option (b)

Question: 3

In the given figure on a circle of radius 7 cm, tangent PT is drawn from a point P such that PT = 24 cm. If O is the centre of the circle, then the length of PR is



(a) 30 cm
(b) 28 cm
(C) 32 cm
(d) 25 cm

Solution:

Clearly PT is a tangent and OT is radius
$\therefore \angle OTP = 90^\circ$
$\Rightarrow \triangle OTP$ is a right angled triangle.
Using Pythagoras Theorem, we get

$$OP^2 = OT^2 + PT^2$$
$$\Rightarrow OP^2 = 7^2 + 24^2 = 625$$
$$\Rightarrow OP = 25 \text{ cm}$$

Now, PR = OP + OR = 25 + 7 = 32
Hence, the correct answer is option (c).

Question: 4

225 can be expressed as
(a) 5×3^2
(b) $5^2 \times 3$
(c) $5^2 \times 3^2$
(d) $5^3 \times 3$

Solution:

225 can be written as:
$225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$
Hence, the correct answer is option (c).

Question: 5

The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is
(a) 4/15
(b) 2/15
(c) 1/15
(d) 1/5

Solution:

The multiples of 4 from 1 to 15 are 4, 8 and 12.
Hence, the probability of selecting a multiple of four $= \frac{3}{15} = \frac{1}{5}$
Hence, the correct answer is option (c).

Question: 6

If one zero of a quadratic polynomial ($kx^2 + 3x + k$) is 2, then the value of k is
(a) 5/6
(b) −5/6
(c) 6/5
(d) −6/5

Solution:

Given: A zero of the quadratic polynomial $p(x) = kx^2 + 3x + k$ is 2.
This implies that
$p(2) = 0$
$\Rightarrow k(2)^2 + 3(2) + k = 0$

$\Rightarrow 4k + 6 + k = 0$
$\Rightarrow 5k = -6$
$\Rightarrow k = -\frac{6}{5}$
Hence, the correct answer is option (d).

Question: 7

$2.\overline{35}$ is

(a) an integer
(b) a rational number
(c) an irrational number
(d) a natural number

Solution:

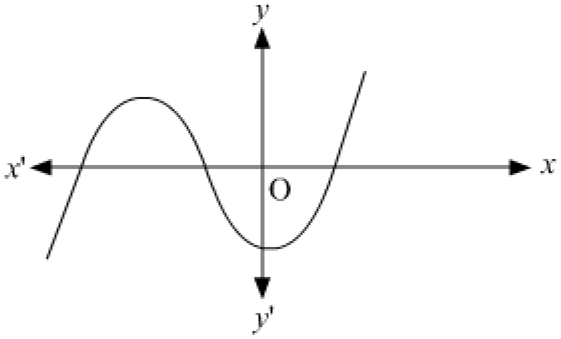
$2.\overline{35} = 2.35353535...$

$2.\overline{35}$ is a non-terminating repeating decimal.

And we know that every non-terminating repeating decimal is a rational number.
Hence, the correct answer is option (b).

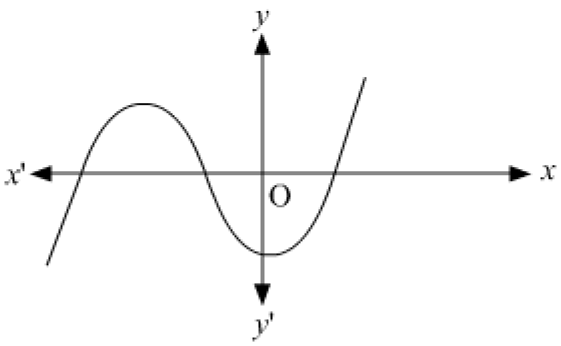
Question: 8

The graph of a polynomial is shown in figure, then the number of its zeroes is



(a) 3
(b) 1
(c) 2
(d) 4

Solution:



The number of zeroes is 3 as the graph of the polynomial cuts the x-axis at 3 points which means the value of y is zero at those points.

Hence, the correct answer is option (a).

Question: 9

Distance of point P(3, 4) from *x*-axis is
(a) 3 units
(b) 4 units
(c) 5 units
(d) 1 unit

Solution:

The perpendicular distance of point (3,4) from *x*-axis is given by its ordinate, i.e. 4.
Hence, the correct answer is option (b).

Question: 10

If the distance between the points A(4, *p*) and B(1, 0) is 5 units, then the value(s) of *p* is (are)
(a) 4 only
(b) −4 only
(c) ± 4
(d) 0

Solution:

The distance between the points A(4, p) and B(1, 0) is given by

$$\sqrt{(4-1)^2+(p-0)^2}$$
$$=\sqrt{3^2+p^2}$$

$= \sqrt{9 + p^2}$

According to the question,

$\sqrt{9 + p^2} = 5$

$\Rightarrow 9 + p^2 = 5^2$

$\Rightarrow p^2 = 25 - 9$

$\Rightarrow p = \pm\sqrt{16}$

$\Rightarrow p = \pm 4$

Hence, the required answer is option (c).

Question: 11

Fill in the blank.

If the point C(*k*, 4) divides the line segment joining two points A(2, 6) and B(5, 1) in ratio 2 : 3, the value of *k* is ___________.

OR

Fill in the blank.

If points A(−3, 12), B(7, 6) and C(*x*, 9) are collinear, then the value of *x* is __________.

Solution:

Using the Section formula, we have

$k = \frac{2 \times 5 + 3 \times 2}{2 + 3}$

$\Rightarrow k = \frac{10 + 6}{5}$

$\Rightarrow k = 3.2$

OR

For collinear points, Area = 0.

i.e.

$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$

$\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right| = 0$

$\left|-3(6 - 9) + 7(9 - 12) + x(12 - 6)\right| = 0$

$\left|-3(-3) + 7(-3) + x(6)\right| = 0$

$\left|9 - 21 + 6x\right| = 0$

$\left|6x - 12\right| = 0$

$\Rightarrow 6x - 12 = 0$

$\Rightarrow 6x = 12$

$\Rightarrow x = 2$

The value of *x* is 2.

Question: 12

Fill in the blank.

If the equations $kx - 2y = 3$ and $3x + y = 5$ represent two intersecting lines at unique point, then the value of k is __________.

OR

Fill in the blank.

If quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of k is __________.

Solution:

Since the given equations represent two lines intersecting at a unique point, they've got a unique solution.

Therefore,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{k}{3} \neq \frac{-2}{1}$

$\Rightarrow k \neq -6$

Hence, the given pair of linear equations in two variables will have a unique solution for all values of k except −6.

OR

Given that the quadratic equation $3x^2 - 4x + k$ has equal roots.

$\Rightarrow D = 0$

$\Rightarrow b^2 - 4ac = 0$

where $a = 3,\ b = -4,\ c = k$

$\Rightarrow (-4)^2 - 4 \times 3 \times k = 0$

$\Rightarrow 16 - 12k = 0$

$\Rightarrow 12k = 16$

$\Rightarrow k = \frac{4}{3}$

⇒ The value of k is $\frac{4}{3}$

Question: 13

The value of (sin 20° cos 70° + sin 70° cos 20°) is __________.

Solution:

The given expression is
$\sin 20^\circ \cos 70^\circ + \sin 70^\circ \cos 20^\circ$
$= \sin 20^\circ \cos (90^\circ - 20^\circ) + \sin (90^\circ - 20^\circ) \cos 20^\circ$
$= \sin 20^\circ \sin 20^\circ + \cos 20^\circ \cos 20^\circ$
$= \sin^2 20^\circ + \cos^2 20^\circ$
$= 1$

Question: 14

Fill in the blank.
If tan (A + B) = $\sqrt{3}$ and tan (A − B) = $\frac{1}{\sqrt{3}}$, A > B, then the value of A is __________.

Solution:

Given, $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$
Therefore, $A + B = 60^\circ$ and $A - B = 30^\circ$
Adding the two equations, we get
$2A = 90^\circ$
$\Rightarrow A = \frac{90^\circ}{2} = 45^\circ$

Question: 15

Fill in the blank.

The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is ___________.

Solution:

We know that, the ratio of the perimeter of two similar triangles = ratio of their corresponding sides

$\Rightarrow \frac{25}{15} = \frac{9}{x}$
$\Rightarrow x = \frac{9 \times 15}{25} = \frac{27}{5}$
$\Rightarrow x = 5.4$
Hence, the corresponding side of the second triangle is 5.4 cm.

Question: 16

If 5tanθ = 3, then what is the value of $\left(\frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta}\right)$?

Solution:

Given that $5 \tan \theta = 3$
$\Rightarrow \tan \theta = \frac{3}{5}$
$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{3}{5}$
Let $\sin \theta = 3k$ and $\cos \theta = 5k$, where k is any integer.

Consider the given expression:
$\frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta}$
$= \frac{5(3k) - 3(5k)}{4(3k) + 3(5k)}$
$= \frac{0}{27k}$
$= 0$

Question: 17

The areas of two circles are in the ratio 9 : 4, then what is the ratio of their circumferences?

Solution:

Given: Ratio of the areas of the two circle is 9 : 4.
Let the areas of the the two circles be A_1 and A_2.
Hence, $\frac{A_1}{A_2} = \frac{9}{4}$ .....(1)
Let the radii of the two circles be r_1 and r_2.
$\frac{\text{Area of the first circle}}{\text{Area of the second circle}} = \frac{\pi {r_1}^2}{\pi {r_2}^2}$
$\Rightarrow \frac{\pi {r_1}^2}{\pi {r_2}^2} = \frac{9}{4}$ [From (1)]
$\Rightarrow \frac{{r_1}^2}{{r_2}^2} = \frac{9}{4}$
$\Rightarrow \frac{r_1}{r_2} = \frac{3}{2}$ .....(2)
Let the circumferences of the two circles be C_1 and C_2.
$\frac{C_1}{C_2} = \frac{2\pi r_1}{2\pi r_2}$
$\Rightarrow \frac{C_1}{C_2} = \frac{r_1}{r_2}$
$\Rightarrow \frac{C_1}{C_2} = \frac{3}{2}$ [From (2)]
The ratio of their circumference is 3 : 2.

Question: 18

If a pair of dice is thrown once, then what is the probability of getting a sum of 8?

Solution:

Total number of outcomes when two dices are thrown simultaneously is given by,

$\text{Total outcomes} = 6 \times 6 = 36$

Favorable pair of outcomes for getting a sum of 8 is given by,

$(2,\ 6),\ (3,\ 5),\ (4,\ 4),\ (5,\ 3),\ (6,\ 2)$

$\Rightarrow \text{Total number of favourable outcomes} = 5$

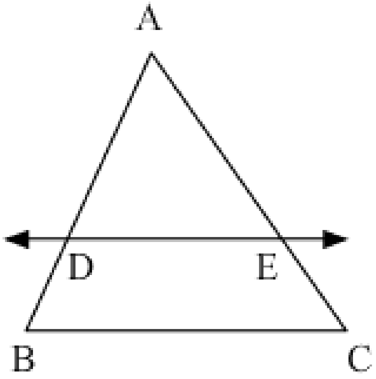
Therefore,

$$\text{Required probability} = \frac{\text{Total number of favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{5}{36}$$

Question: 19

In the given figure in ΔABC, DE || BC such that AD = 2.4 cm, AB = 3.2 cm and AC = 8 cm, then what is the length of AE?



Solution:

In △ABC, DE is given to be parallel to BC.

Therefore, using BPT, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{AB-AD} = \frac{AE}{AC-AE}$$

$$\Rightarrow \frac{2.4}{3.2-2.4} = \frac{AE}{8-AE}$$

$\Rightarrow 3=\frac{\mathrm{AE}}{8-\mathrm{AE}}$
$\Rightarrow 24-3\mathrm{AE}=\mathrm{AE}$
$\Rightarrow 4\mathrm{AE}=24$
$\Rightarrow \mathrm{AE}=6\ \mathrm{cm}$

Question: 20

The n^{th} term of an AP is (7 – 4*n*), then what is its common difference?

Solution:

Given: n^{th} term of an AP is $(7-4n)$
Since n^{th} term of an AP is given by
$T_{\mathrm{n}}=a+(n-1)d,\ \text{where } a=\text{First Term},\ d=\text{Common Difference.}$

Therefore, we have
$7-4n=a+(n-1)d$
$\Rightarrow 7-4n=(a-d)+nd$
Comparing both sides, we get
$d=-4$
Hence, the common difference is -4.

Question: 21

A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag.

Solution:

Given, the number of red balls in the bag = 5
Let the number of blue balls in the bag = *x*

Therefore, the total number of balls in the bag = *x* + 5

Let R and B denote the events of drawing a red ball and a blue ball respectively from the bag.

Then, according to the question, we have

$P(B)=3P(R)$
$\Rightarrow \frac{x}{x+5}=3\times\frac{5}{x+5}$

$\Rightarrow \frac{x}{x+5} = \frac{15}{x+5}$

$\Rightarrow x = 15$

Hence, the number of blue balls in the bag is 15.

Question: 22

Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$.

OR

Prove that $\frac{\tan^2\ \theta}{1+\tan^2\ \theta} + \frac{\cot^2\ \theta}{1+\cot^2\ \theta} = 1$

Solution:

Consider the LHS:

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}}$

$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \sec\theta - \tan\theta$

=RHS

Hence proved.

OR

Consider the LHS:

$\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$

$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\frac{1}{\tan^2\theta}}{1+\frac{1}{\tan^2\theta}}$

$$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\frac{1}{\tan^2\theta}}{\frac{1+\tan^2\theta}{\tan^2\theta}}$$

$$= \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{1}{1+\tan^2\theta}$$

$$= \frac{1+\tan^2\theta}{1+\tan^2\theta}$$

$$= 1$$

=RHS

Question: 23

Two different dice are thrown together, find the probability that the sum a of the numbers appeared is less than 5.

OR

Find the probability that 5 Sundays occur in the month of November of a randomly selected year.

Solution:

Two dices are thrown simultaneously.
So, the total number of outcomes will be $6^2 = 36$.

Now, all the favorable pairs whose sum is less than 5 is given by (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1).

The total number of favorable outcomes is 6.

Hence, the required probability = 6/36 = 1/6.

OR

In any randomly selected year, the Month of November will have 30 days.

Now out of these 30 days, we will have 4 complete weeks (i.e. 28 days) having 4 Sundays.

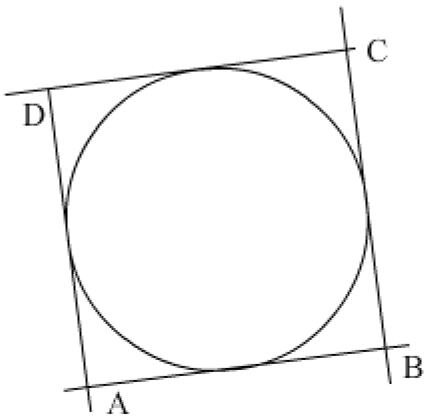
For the remaining two days, we have the following possibilities:
(i) Saturday and Sunday,
(ii) Sunday and Monday,
(iii) Monday and Tuesday,
(iv) Tuesday and Wednesday,
(v) Wednesday and Thursday,
(vi) Thursday and Friday,

(vii) Friday and Saturday.

Thus, the possibility of having a 5th Sunday = 2/7.

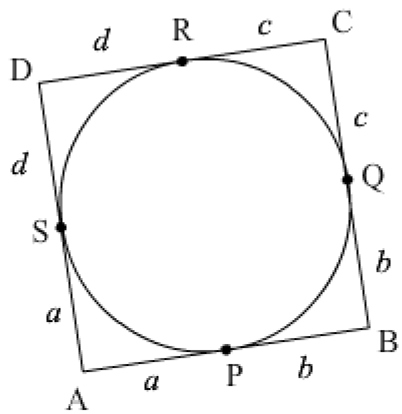
Question: 24

In the given figure, a circle touches all the four sides of a quadrilateral ABCD. If AB = 6 cm, BC = 9 cm and CD = 8 cm, then find the length of AD.



Solution:

The figure given in the question is below.



From the property of tangents we know that, the length of two tangents drawn from the same external point will be equal. Therefore we have the following:

SA = AP, PB = BQ, QC = CR, DR = DS.

For our convenience, let us
represent SA and AP by *a*, PB and BQ by *b*, QC and CR by *c* and DR and DS by *d*.

It is given in the problem that, AB = 6
Observing the figure, we can rewrite the above equation as follows:

AP + PB = 6
⇒ $a + b = 6$
⇒$b = 6 - a$

BC = 9(Given)
Since, BQ + QC=BC

Therefore,
⇒ BQ + QC = 9
⇒ $b + c = 9$
⇒ $6 - a + c = 9$
⇒ $c = a + 3$

Also, CD = 8(Given)
⇒ CR + RD = 8
⇒ $c + d = 8$
⇒ $a + 3 + d = 8$
⇒ $a + d = 5$

As per our representations, we can write the above equation as follows:
SA + DS = 5

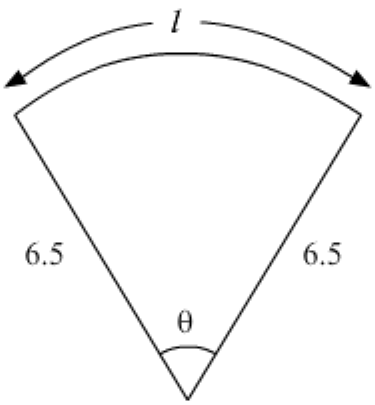
Observing the figure, we can write the following,
SA + DS =AD

Comparing with the above equation we found that the length of side AD is 5 cm.

Question: 25

The perimeter of a sector of a circle with radius 6.5 cm is 31 cm, then find the area of the sector.

Solution:



The given sector of a circle has radius (r) = 6.5 cm
Its perimeter is given to be 31 cm.

$$\Rightarrow 2r + l = 31$$
$$\Rightarrow 2(6.5) + l = 31$$
$$\Rightarrow l = 18$$

Now, the area of the sector is given by
$$\frac{1}{2} \times r \times l$$
$$= \frac{1}{2} \times \frac{13}{2} \times 18$$
$$= 58.5 \ \text{cm}^2$$

Question: 26

Divide the polynomial ($4x^2 + 4x + 5$) by ($2x + 1$) and write the quotient and the remainder.

Solution:

We can do the division as follows:

$$\begin{array}{r|l} & 2x+1 \\ \hline 2x+1 & 4x^2+4x+5 \\ & \underline{4x^2+2x} \\ & \quad\ \ 2x+5 \\ & \quad\ \ \underline{2x+2} \\ & \quad\quad\quad 4 \end{array}$$

The quotient obtained is $2x + 1$ and the remainder is 4.

Question: 27

If α and β are the zeros of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$.

Solution:

The zeros of the polynomial $f(x) = x^2 - 4x - 5$ are given to be α and β.
We have,
$\alpha + \beta = -\frac{-4}{1} = 4 \text{ and } \alpha\beta = \frac{-5}{1} = -5$
Using the identity $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$, we have

$$(4)^2 = \alpha^2 + \beta^2 + 2(-5)$$
$$\Rightarrow \alpha^2 + \beta^2 = 16 + 10 = 26$$

Question: 28

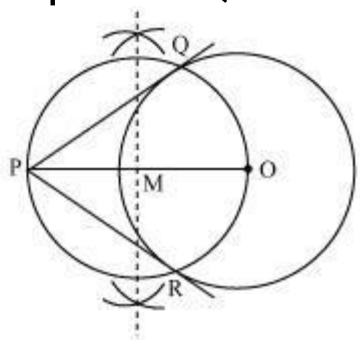
Draw a circle of radius 4 cm. From a point 7 cm away from the centre of circle. Construct a pair of tangents to the circle.

OR

Draw a line segment of 6 cm and divide it in the ratio 3 : 2.

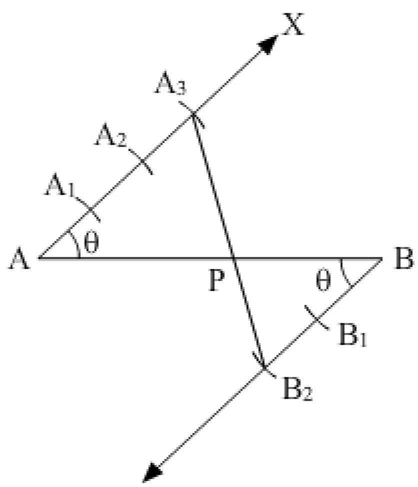
Solution:

A pair of tangents to the given circle can be constructed as follows.
Step 1 Draw a circle of 4 cm radius and O as centre. Locate a point P, 7 cm away from O. Join OP.
Step 2 Bisect OP. Let M be the mid-point of PO.
Step 3 Taking M as centre and MO as radius, draw a circle.
Step 4 Let this circle intersect the previous circle at point Q and R.
Step 5 Join PQ and PR. PQ and PR are the required tangents.



OR

We need to follow the following steps to construct the given



Step of construction
Step I- First of all we draw a line segment AB = 6 cm.

Step II- We draw a ray AX making an acute angle with AB.
Step III- Draw a ray BY parallel to AX by making an acute angle ∠ABY = ∠BAX.

Step IV- Mark three points A_1, A_2, A_3 on AX and two points B_1, B_2 on BY in such a way that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$.

Step V- Join A_3B_2. This line intersects AB at a point P.

Thus, the given line segment AB has been divided internally in the ratio of 3 : 2

Question: 29

A solid metallic cuboid of dimension 24 cm × 11 cm × 7 cm is melted and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed.

Solution:

The dimensions of solid metallic cuboid are given to be 24 cm $\times$ 11 cm $\times$ 7 cm.
The volume of the cuboid = $24 \times 11 \times 7 = 1848\ \text{cm}^3$

Now, it is melted and recast into solid cones of base radius (r) = 3.5 cm and height (h) = 6 cm.
The volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{7\times7}{2\times2} \times 6 = 77\ \text{cm}^3$
Let n be the number of such cones formed.

Now, the volume of the cuboid = The volume of n cones
$1848 = 77n$
$\Rightarrow 77n = 1848$
$\Rightarrow n = \frac{1848}{77} = 24$
Therefore, 24 such cones will be formed.

Question: 30

Prove that (1 + tan A − sec A) × (1 + tan A + sec A) = 2 tan A

OR

Prove that $\frac{\text{cosec }\theta}{\text{cosec }\theta-1} + \frac{\text{cosec }\theta}{\text{cosec }\theta+1} = 2\sec^2\theta$

Solution:

Consider the LHS:

$$(1+\tan A-\sec A)(1+\tan A+\sec A)$$
$$=[(1+\tan A)-\sec A][(1+\tan A)+\sec A]$$
$$=(1+\tan A)^2-\sec^2 A$$
$$=1+2\tan A+\tan^2 A-\sec^2 A$$
$$=1+2\tan A+(-1)$$
$$=2\tan A$$

=RHS

Hence proved.

OR

LHS

$$=\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-1}+\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta+1}$$
$$=\frac{\operatorname{cosec}\theta(\operatorname{cosec}\theta+1)+\operatorname{cosec}\theta(\operatorname{cosec}\theta-1)}{\operatorname{cosec}^2\theta-1}$$
$$=\frac{\operatorname{cosec}^2\theta+\operatorname{cosec}\theta+\operatorname{cosec}^2\theta-\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-1}$$
$$=\frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}\ \left(\because \operatorname{cosec}^2\theta-1=\cot^2\theta\right)$$
$$=\frac{2}{\sin^2\theta}\times\frac{\sin^2\theta}{\cos^2\theta}\ \left(\because \operatorname{cosec}^2\theta=\frac{1}{\sin^2\theta}\ \text{and}\ \cot^2\theta=\frac{\cos^2\theta}{\sin^2\theta}\right)$$
$$=\frac{2}{\cos^2\theta}$$
$$=2\sec^2\theta\ \left(\because\ \sec^2\theta=\frac{1}{\cos^2\theta}\right)$$
$$=\text{RHS}$$

Hence proved.

Question: 31

Given that √3 is an irrational number, show that (5 + 2√3) is an irrational number.

OR

An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Given: $\sqrt{3}$ is an irrational number.
To prove: $5+2\sqrt{3}$ is an irrational number.
Proof:
Suppose $5+2\sqrt{3}$ is a rational number.
Therefore it can be written in $\frac{p}{q}$ form, where p and q are coprime integers.

$\Rightarrow 5+2\sqrt{3}=\frac{p}{q}$

$\Rightarrow 2\sqrt{3}=\frac{p}{q}-5=\frac{p-5q}{q}$

$\Rightarrow \sqrt{3}=\frac{p-5q}{2q}$

Since p and q are integers, therefore $\frac{p-5q}{2q}$ must be a rational number.
But this is a contradiction as the LHS is an irrational number.
Our supposition was wrong.
Hence, $5+2\sqrt{3}$ is an irrational number.
Hence proved.

OR

We are given that an army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. We need to find the maximum number of columns in which they can march.

Members in army = 612
Members in band = 48.
Therefore,

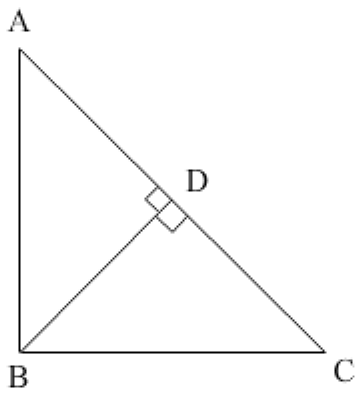
The maximum number of columns = H.C.F of 612 and 48.
By applying Euclid's division lemma

$612 = 48 \times 12 + 36$
$48 = 36 \times 1 + 12$
$36 = 12 \times 3 + 0$
Therefore, H.C.F. = 12
Hence, the maximum number of columns in which they can march is 12.

Question: 32

Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:



The given triangle is right-angled at B.

We need to prove that the square of the hypotenuse is equal to the sum of the squares of the other two sides i.e. $AC^2 = AB^2 + BC^2$.

Construction $BD \perp AC$

In ΔABD and ΔABC:
∠ADB = ∠ABC = 90°
∠BAD = ∠BAC (Common)
∴ ΔADB ~ ΔABC (By AA similarity criterion)

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$$
$$\Rightarrow AD \times AC = AB^2 \qquad .....(1)$$

In ΔBDC and ΔABC:
∠BDC = ∠ABC = 90°
∠BCD = ∠BCA (Common)
∴ ΔBDC ~ ΔABC (By AA similarity criterion)

$$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC}$$
$$\Rightarrow CD \times AC = BC^2 \qquad .....(2)$$

Adding (1) and (2), we get

$$\begin{aligned} AB^2 + BC^2 &= AD \times AC + CD \times AC \\ &= (AD + CD) \times AC \\ &= AC \times AC \\ &= AC^2 \end{aligned}$$

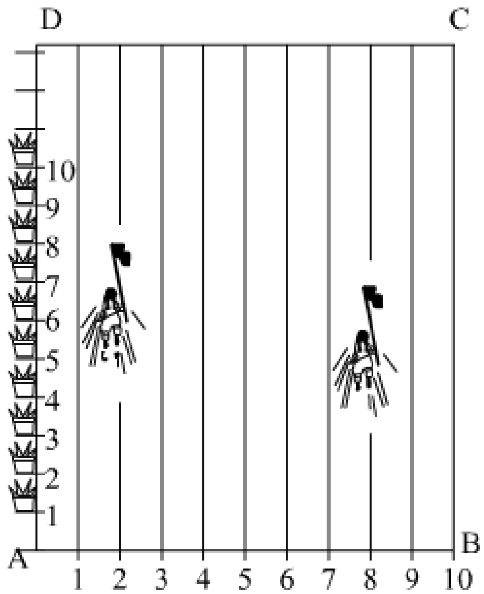
Hence proved.

Question: 33

Read the following passage carefully and then answer the questions given at the end.

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each.

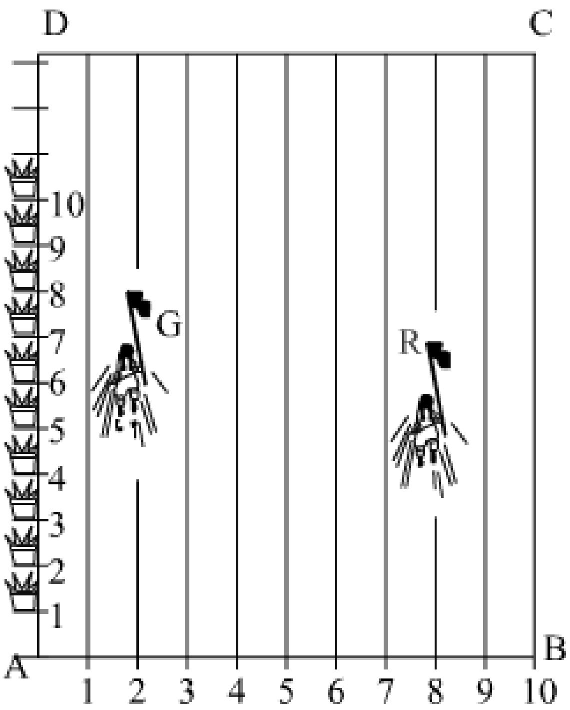
100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Figure. Niharika runs 1/4th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5th the distance AD on the eighth line and posts a red flag.



(i) What is the distance between the two flags?

(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag?

Solution:



It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\left(\frac{1}{4}\times 100\right)\text{m} = 25\text{m}$ from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e., $\left(\frac{1}{5}\times 100\right)\text{m} = 20\text{m}$ from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

(i) Distance between these flags by using distance formula = GR

$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61}\text{ m}$

(ii) The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (*x*, *y*).

$x = \frac{2+8}{2}, \quad y = \frac{25+20}{2}$

$x = \frac{10}{2} = 5, \quad y = \frac{45}{2} = 22.5$

Hence, $A(x, y) = (5, 22.5)$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line.

Question: 34

Solve graphically: $2x + 3y = 2$, $x - 2y = 8$

Solution:

Given:
$2x + 3y = 2$
$x - 2y = 8$

Consider the first equation $2x + 3y = 2$.

x	4	1
y	-2	0

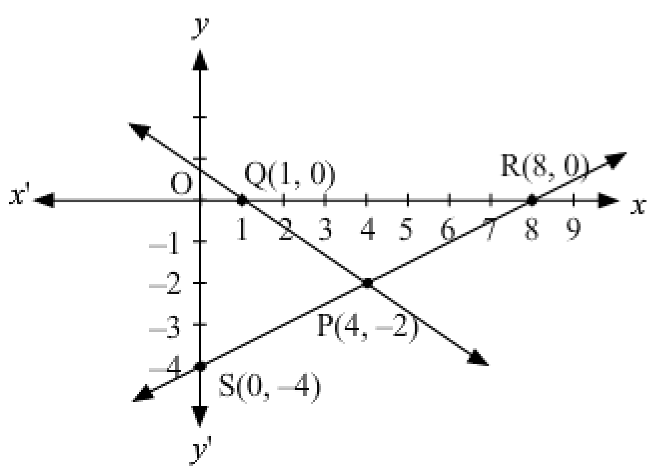
Hence, the points are $\mathrm{P}\left(4,\ -2\right)$ and $\mathrm{Q}\left(1,\ 0\right)$ respectively.

Considering the second equation $x - 2y = 8$.

x	0	8
y	-4	0

Hence, the points are $\mathrm{S}\left(0,\ -4\right)$ and $\mathrm{R}\left(8,\ 0\right)$ respectively.

Now drawing the two lines on the coordinate axis, we get



Hence, the point of intersection of these two line is $\mathrm{P}\left(4,\ -2\right)$.

Question: 35

A two digit number is such that the product of its digits is 14. If 45 is added to the number; the digits interchange their places. Find the number.

Solution:

Let the digits at units and tens places be x and y, respectively.
$\therefore xy = 14$
$\Rightarrow y = \frac{14}{x}$ ... (i)
According to the question:

$$(10y+x)+45=10x+y$$
$$\Rightarrow 9y-9x=-45$$
$$\Rightarrow y-x=-5 \qquad \dots \text{(ii)}$$
From (i) and (ii), we get:
$$\frac{14}{x}-x=-5$$
$$\Rightarrow \frac{14-x^2}{x}=-5$$
$$\Rightarrow 14-x^2=-5x$$
$$\Rightarrow x^2-5x-14=0$$
$$\Rightarrow x^2-\left(7-2\right)x-14=0$$
$$\Rightarrow x^2-7x+2x-14=0$$
$$\Rightarrow x\left(x-7\right)+2\left(x-7\right)=0$$
$$\Rightarrow \left(x-7\right)\left(x+2\right)=0$$
$$\Rightarrow x-7=0 \text{ or } x+2=0$$
$$\Rightarrow x=7 \text{ or } x=-2$$
$$\Rightarrow x=7 \quad (\because \text{ the digit cannot be negative})$$
Putting $x=7$ in equation (i), we get:
$$y=2$$
$$\therefore \text{Required number}=10\times 2+7=27$$

Question: 36

If 4 times the 4th term of an AP is equal to 18 times the 18th term, then find the 22nd term.

OR

How many terms of the AP : 24, 21, 18, ... must be taken so that their sum is 78?

Solution:

Let the first term be a and the common difference be d.
According to question,
$$4a_4=18a_{18}$$
$$\Rightarrow 4\left(a+3d\right)=18\left(a+17d\right)$$
$$\Rightarrow 4a+12d=18a+306d$$
$$\Rightarrow -14a=294d$$
$$\Rightarrow a=-\frac{294}{14}d=-21d$$
22nd term = $a_{22}=a+21d=-21d+21d=0$

OR

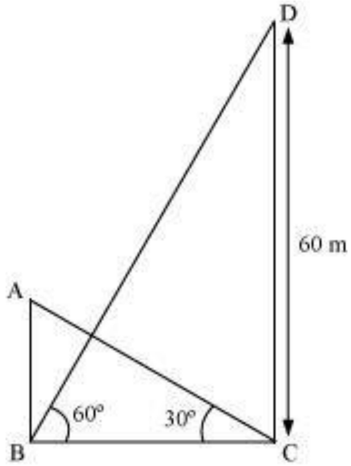
Given that 24, 21, 18, ... is an AP.
Let n terms should be taken to make the sum 78 then
24 + 21 + 18 + ...to n terms = 78
Here $a = 24,\ d = -3,\ S_n = 78,\ n = ?$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$\Rightarrow 78 = \frac{n}{2}\{2 \times 24 + (n-1)(-3)\}$$
$$\Rightarrow 156 = n(48 - 3n + 3)$$
$$\Rightarrow 156 = n(51 - 3n)$$
$$\Rightarrow 3n^2 - 51n + 156 = 0$$
$$\Rightarrow 3\left(n^2 - 17n + 52\right) = 0$$
$$\Rightarrow (n-13)(n-4) = 0$$
$$\therefore\ n = 4,\ 13$$

Question: 37

The angle of elevation of the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

Solution:



Let AB denote the building and CD denote the tower. The height of the tower is given to be 60 m.
∴ CD = 60 m

We have to find the height of the building i.e. AB.
In ΔBCD,

$$\tan 60° = \frac{DC}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60\,m}{BC}$$

$$\Rightarrow BC = \frac{60}{\sqrt{3}}m \qquad (1)$$

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{\frac{60}{\sqrt{3}}m} \qquad \text{[From (1)]}$$

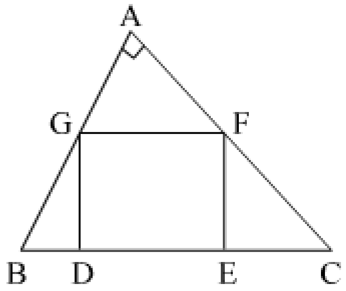
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\,AB}{60\ m}$$

$$\Rightarrow AB = \frac{1}{\sqrt{3}} \times \frac{60\ m}{\sqrt{3}} = \frac{60\ m}{3} = 20\ m$$

Thus, the height of the building is 20 m.

Question: 38

In the given figure, DEFG is a square in a triangle ABC right angled at A.



Prove that
(i) ΔAGF ~ ΔDBG
(ii) ΔAGF ~ ΔEFC

OR

In an obtuse ΔABC (∠B is obtuse), AD is perpendicular to CB produced. Then prove that $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

Solution:

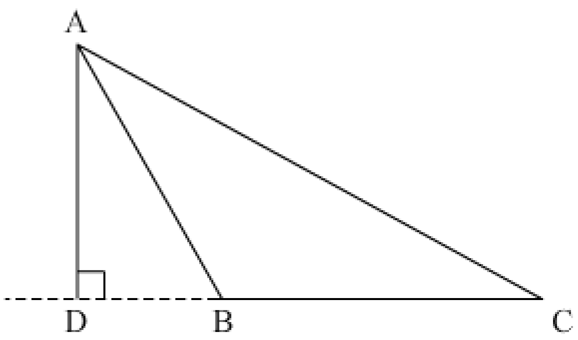
Given: DEFG is a square inside a triangle ABC right angles at A.

(i) In ΔAGF and ΔDBG,
$\angle AGF = \angle GBD$ (Corresponding angles as $GF \parallel BC$)
$\angle GAF = \angle GDB$ (both right angles)
Therefore, $\Delta AGF \sim \Delta DBG$ (By AA similarity theorem)

(ii) In ΔAGF and ΔEFC,
$\angle AFG = \angle FCE$ (Corresponding angles as $GF \parallel BC$)
$\angle GAF = \angle FEC$ (both right angles)
Therefore, $\Delta AGF \sim \Delta EFC$ (By AA similarity theorem)
Hence proved.

OR

Following is the ΔABC, where $\angle B$ is obtuse. AD is perpendicular to CB produced.



In ΔADB, $AB^2 = AD^2 + DB^2$
$\Rightarrow AD^2 = AB^2 - DB^2$ .....(1)
In ΔADC, $AC^2 = AD^2 + DC^2$
$= AB^2 - DB^2 + DC^2$ .....[from (1)]
$= AB^2 + DC^2 - DB^2$
$= AB^2 + (DB + BC)^2 - DB^2$
$= AB^2 + DB^2 + BC^2 + 2\,BC \times BD - DB^2$
$= AB^2 + BC^2 + 2\,BC \times BD$
Hence proved.

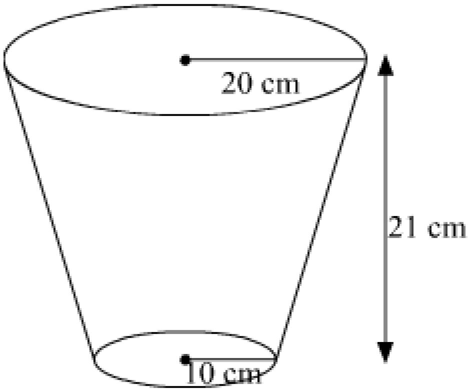
Question: 39

An open metal bucket is in the shape of a frustum of cone of height 21 cm with radii of its lower and upper ends are 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹ 40 per litre.

OR

A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Solution:

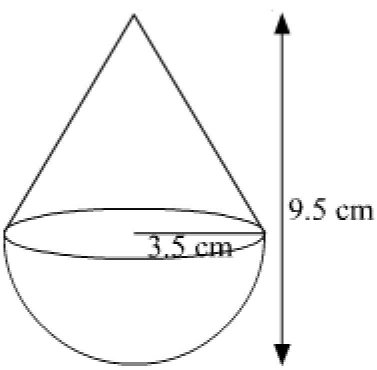


Given, $r_1 = 10\ cm$, $r_2 = 20\ cm$ and $h = 21\ cm$
Therefore, the volume of the bucket (frustum)

$$= \frac{1}{3}\pi h\left(r_1{}^2 + r_2{}^2 + r_1 r_2\right)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times \left(10^2 + 20^2 + 10 \times 20\right)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 700$$
$$= 15400\ cm^3$$
$$= \frac{15400}{1000}\ L$$
$$= 15.4\ L$$

Therefore, the cost of milk $= Rs\ 15.4 \times 40 = Rs\ 616$

OR



Radius of the cone = Radius of the hemisphere = r (say) = 3.5 cm

Therefore, height of the cone = (9.5 − 3.5) = 6 cm (as the height of a hemisphere is equal to it's radius)

Therefore, Volume of the solid = Volume of cone + Volume of hemisphere

Question: 40

Find the mean of the following data :

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	20	35	52	44	38	31

Solution:

Following is the given data:

Classes	f_i	x_i	$f_i x_i$
0-20	20	10	200
20-40	35	30	1050
40-60	52	50	2600
60-80	44	70	3080
80-100	38	90	3420
100-120	31	110	3410
	220		13,760

Therefore, mean = $\frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{13760}{220} = 62.55$